P.G. sem-2nd
अक्टूबर paper-VI, unit-1st
Bilinear # transformation

Remark ① $w = \beta z$ (β may be real or complex) is called dilation

सोम 11 मंगल 12 बुध 13

(IV) $w = \frac{1}{z}$ (Inversion)

(V) Linear Transformation
$w = az + b$, where a & b are complex constants

(VI) $w = f(\bar{z}) = f(z)$ (isogonal)

Theorem 2. Every bilinear transformation is the resultant of bilinear transformations with simple geometric imports.

Proof. Consider a bilinear transformation

$$w = \frac{az+b}{cz+d} \quad \text{--- (1)}$$

where $ad - bc \neq 0$, $c \neq 0$.

From (1)
$$w = \frac{a}{c} \cdot \frac{z+\frac{b}{a}}{z+\frac{d}{c}}$$

$$= \frac{a}{c}\left[1 + \frac{\frac{b}{a}-\frac{d}{c}}{z+\frac{d}{c}}\right]$$

$$= \frac{a}{c} + \frac{bc-ad}{c^2} \cdot \frac{1}{z+\frac{d}{c}}$$

Taking $z_1 = z + \frac{d}{c}$

$z_2 = \frac{1}{z_1}$

$z_3 = \frac{bc-ad}{c^2} z_2$

अक्टूबर 2004						
रवि	सोम	मंगल	बुध	गुरु	शुक्र	शनि
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

e obtain

$$w = \frac{a}{c} + \frac{bc-ad}{c^2}\frac{1}{z_1}$$

$$= \frac{a}{c} + \frac{bc-ad}{c^2}\cdot z_2$$

$$= \frac{a}{c} + z_3 .$$

hich is similar to $z_1 = z + \frac{d}{c}$.

he above three auxiliary transformations, namely z_1, z_2, z_3 are of the form

$$w = z + \alpha$$

$$w = \frac{1}{z}$$

$$w = \beta z$$

This proves that every general bilinear transformation is the resultant of the bilinear transformations

Definition :- The points which coincide with their transformations are called invariant or fixed points of the transformation, that is to say, fixed points of a transformation $w = f(z)$ are obtained by the equation $z = f(z)$

See there

The invariant (or fixed) points of the transformation

$$w = \frac{az+b}{cz+d} \quad \text{——— (1)}$$

are given by

$$z = \frac{az+b}{cz+d}$$

$$\text{or } cz^2 - (a-d)z - b = 0$$

$$\text{or } z = \frac{(a-d) \pm \sqrt{(a-d)^2 + 4bc}}{2c}$$

$$\text{or } z = \frac{(a-d) \pm \sqrt{M}}{2c} \quad \text{——— (2)}$$

where $M = (a-d)^2 + 4bc$.

The number of finite fixed points is one or two according as $M = 0$ or $M \neq 0$

Case I. Let $c = 0$ and $d \neq 0$

Then (1) becomes $w = \frac{az+b}{0+d} = \frac{a}{d}z + \frac{b}{d}$

The fixed point is given by

$$z = \frac{a}{d}z + \frac{b}{d} \quad \text{or} \quad z = \frac{b}{d-a} \quad \text{—— (3)}$$

Now if $a-d \neq 0$, then (2) declares that one fixed and (3) declares that the other fixed point is finite.

If $a-d=0$, then the transformation has one fixed point i.e, ∞ according to (3). Thus we have the following result

अक्टूबर 2004

रवि	सोम	मंगल	बुध	गुरु	शुक्र	शनि
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

Thus we have the following results:

(I) If $c \neq 0$, and $M \neq 0$, there are two finite fixed points

(II) If $c \neq 0$ and $M = 0$, one finite fixed point

(III) If $c = 0$ and $a - d = 0$, only one fixed point, i.e., ∞

In this case $w = z + \frac{b}{d}$

(IV) If $c = 0$, $a - d \neq 0$, one finite and other is ∞

Ex. Invariant points of $w = z^3$ are the solutions of the equation $z = z^3$